DUSO Mathematics League 2016 - 2017

Contest #5. **Calculators are not permitted on this contest.**

Part I. **ALGEBRA I** *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

5 - 1. The quadratic polynomial $4x^2 - 25$ factors into two linear factors with integer coefficients. Compute the sum of those linear factors.

5 - 2. In a survey of teenagers, $\dfrac{2}{3}$ of them like jazz music, $\dfrac{3}{4}$ of them like country music, and $\dfrac{4}{5}$ of them like pop music. Compute the least fraction of these teenagers that could possibly like all three kinds of music.

Part II. **GEOMETRY** *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

5 - 3. Triangle ABC has sides of length $AB = 3$, $BC = 5$, and $AC = 7$. After a dilation, the image of $\triangle ABC$ is $\triangle DEF$. If $EF = 105$, compute DF.

5 - 4. Points A and B have coordinates $A(-2, 4)$ and $B(4, 2)$. Suppose that point P is somewhere on the x-axis, at $(p, 0)$, such that the points A, P, and B are on a circle tangent to the x-axis. Compute p.

Part III. **ALGEBRA II / ADVANCED TOPICS** *Time Limit: 10 minutes*

The word "compute" calls for an exact answer in simplest form.

5 - 5. If $i = \sqrt{-1}$, compute the value of $(3 - 2i)^3$ in the form $a + bi$.

5 - 6. Let A represent the sequence $3, \dfrac{3}{2}, \dfrac{3}{4}, \cdots$, with $a_k = 3\left(\dfrac{1}{2}\right)^{k-1}$. Define a sequence B such that $b_k = a_k^2 + a_k$. Compute the sum of all the terms of B.

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R-1. Compute the smallest odd positive integer that is the product of three distinct prime numbers.

R-2. *Let N be the number you will receive.* The quadratic equation $x^2 - 8x - N = 0$ has two roots. Compute the greater of these two roots.

R-3. *Let N be the number you will receive.* An arithmetic sequence begins $18, N, \cdots$. The difference between any two consecutive terms is constant. Compute the fifth term in the sequence.

R-4. *Let N be the number you will receive.* A cylindrical can with a closed top and closed bottom has a surface area of $N\pi$ square cm and a base radius of 1 cm. Compute the volume of the can in cubic cm.

R-5. *Let N be the number you will receive.* The lateral area of a right circular cone is $N/2$. The height of the cone is $\dfrac{\sqrt{15}}{2}$. Compute the radius of the cone.